

Subspace: $U \subset V$ is a subspace if i) $0 \in U$ (additive id.)

ii) $\alpha, \beta \in F, x, y \in U, \alpha x + \beta y \in U$

eg i) $S = \{ (x, y, z) : x + y = 1 \}$ subspace ?? $\mathbb{R}^3$

$$X \in S \quad X = (x_1, y_1, z_1) \quad x_1 + y_1 = 1 \quad \alpha, \beta \in F$$

$$Y \in S \quad Y = (x_2, y_2, z_2) \quad x_2 + y_2 = 1$$

$\alpha X + \beta Y \in S$ (To show) (not a subspace)

$$\alpha (x_1, y_1, z_1) + \beta (x_2, y_2, z_2)$$

$$= (\alpha x_1, \alpha y_1, \alpha z_1) + (\beta x_2, \beta y_2, \beta z_2)$$

$$= (\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2, \alpha z_1 + \beta z_2)$$

$$\alpha x_1 + \beta x_2 + \alpha y_1 + \beta y_2 = 1 \quad ?? \text{ check } \times$$

$$\alpha (x_1 + y_1) + \beta (x_2 + y_2) = \alpha \cdot 1 + \beta \cdot 1 = \alpha + \beta \neq 1$$

$$2) \quad T = \{ (x, y, z) : x = 5y \}$$

$$\left. \begin{array}{l} X \in T \quad (x_1, y_1, z_1) \quad x_1 = 5y_1 \\ Y \in T \quad (x_2, y_2, z_2) \quad x_2 = 5y_2 \end{array} \right\}$$

$$\alpha X + \beta Y \in T$$

$$\alpha X + \beta Y = (\alpha x_1 + \beta x_2, \alpha y_1 + \beta y_2, \alpha z_1 + \beta z_2)$$

$$\text{To prove: } \alpha x_1 + \beta x_2 = 5(\alpha y_1 + \beta y_2)$$

$\Downarrow$

$$\alpha \cdot 5y_1 + \beta \cdot 5y_2$$

$$= 5\alpha y_1 + 5\beta y_2$$

$$= 5(\alpha y_1 + \beta y_2)$$

T forms a subspace

3) $U = \{ (x, y, z) : \underline{x = z^2} \}$ subspace $\checkmark$

$x \in U : (x, y, z) \quad x = z^2$

$y \in U : (x_2, y_2, z_2) \quad x_2 = z_2^2$

$\alpha x + \beta y \in U$ To prove: $\alpha x + \beta y = (\alpha z + \beta z_2)^2$

$\alpha z_1^2 + \beta z_2^2 = \alpha^2 z_1^2 + \beta^2 z_2^2 + 2\alpha\beta z_1 z_2$
is not true

T: Not a subspace

Eg: Can you write $(1, 2, 3)$ in terms of $(1, 0, 3)$ & $(-1, 1, -3)$

To find α_1, α_2

$(1, 2, 3) = \alpha_1 (1, 0, 3) + \alpha_2 (-1, 1, -3)$

$(1, 2, 3) = (\alpha_1 - \alpha_2, \alpha_2, 3\alpha_1 - 3\alpha_2)$

$\alpha_1 - \alpha_2 = 1, \quad \underline{\alpha_2 = 2}, \quad 3\alpha_1 - 3\alpha_2 = 3$

$\underline{\alpha_1 = 3}$

$3 \cdot 3 - 3 \cdot 2 = 9 - 6 = 3$

$\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = 0$

$\alpha_1 = 3, \quad \alpha_2 = 2, \quad \alpha_3 = -1$

If $\alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_n x_n = 0$ for $\alpha_1, \alpha_2, \dots, \alpha_n \in F$

and all α_i 's are not zero.

then set $\{x_1, x_2, \dots, x_n\}$ is called linearly dependent.

Think of $S = \{ (1, 2, 3), (1, 0, 3), (-1, 1, -3) \}$

Eg 2) $S = \{ (1, 0), (0, 1) \}$

$$(2, 3) = 2(1, 0) + 3(0, 1)$$

$\mathbb{R}^2$ can be generated from S

def: Let S is called span of V if every element

V can be generated using S or

$$x = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \dots + \alpha_n x_n$$

$$\text{Span}(S) = V$$